Contents

CHAPTER 1

CHAPTER

1 Prerequisites for Calculus

1.1 Linear Functions

1.2 Functions and Graphs

1.3 Exponential Functions

1.4 Parametric Equations

1.5 Inverse Functions and Logarithms

1.6 Trigonometric Functions

Willard Libby won the 1960 Nobel Prize in Chemistry for his 1949 discovery of the method of **radiocarbon dating** for estimating the age of organic substances that were once living tissue. The method relies on the slow decay of carbon-14, a radioactive isotope of carbon with a half-life of approximately 5730 years. Using exponential functions and the properties reviewed in Section 1.3, researchers have used radiocarbon dating to study the carefully preserved remains of early Egyptian dynasties and thus reconstruct the history of an important ancient civilization.

CHAPTER 1 Overview

The main prerequisite for a student who wants to undertake the study of calculus is an understanding of functions. It is the context of functions that brings coherence to the study of algebra and provides the connection between algebra and geometry, especially through the graphical representations of algebraic expressions. Students who have taken a modern precalculus course (emphasizing algebraic, numerical, and graphical representations of functions) will probably have already seen everything in this first chapter, but we offer it here mainly in the spirit of review. Since calculus is basically a tool for understanding how functions of various kinds model real-world behavior, a solid understanding of the basic functions makes the applications of calculus considerably easier. Any time you spend in this chapter strengthening your understanding of functions and graphs will pay off later in the course.

We begin by reviewing the easiest type of function, surely the most familiar to most students. Happily, it is also the most important (by far) for understanding how calculus works.

1.1 Linear Functions

You will be able to analyze linear functions in their algebraic, numerical, and graphical representations.

- Increments and slope
- Linear models
- Point-slope form of linear equations
- Other forms of linear equations
- Parallel and perpendicular lines
- Simultaneous linear equations

Increments and Slope

Calculus is the mathematics of change. It explores the fundamental question: When two quantities are linked, how does change in one affect change in the other?

Calculus is written in the language of functions, $y = f(x)$, because functions describe how one quantity or variable depends on another. The distance a person has walked can be a function of time. The pressure that an underwater diver feels is a function of depth below the surface. Calculus asks: If we know how much time has passed, can we determine how far the person has walked? If we know how far the diver has descended, do we also know how much the pressure has increased? These changes in time, distance, or pressure are known as *increments*.

DEFINITION Increments

The change in a variable, such as t from t_1 to t_2, x from x_1 to x_2, or y from y_1 to y_2, is called an **increment** in that variable, denoted by

$$\Delta t = t_2 - t_1, \quad \Delta x = x_2 - x_1, \quad \Delta y = y_2 - y_1.$$

The symbols Δt, Δx, and Δy are read "delta t," "delta x," and "delta y." The letter Δ is a Greek capital D for "difference." Neither Δt, Δx, nor Δy denotes multiplication; Δx is not "delta times x." Increments can be positive, negative, or zero.

EXAMPLE 1 Finding Increments

(a) Find the increments Δx and Δy from the point $(1, -3)$ to the point $(4, 5)$. Find the increments Δx and Δy from the point $(5, 6)$ to the point $(5, 1)$.

(b) As a diver descends below sea level from a depth of $d_1 = 20$ meters to $d_2 = 25$ meters, the pressure increases from $p_1 = 2.988$ to $p_2 = 3.485$ atmospheres. What are the increments in depth, Δd, and pressure, Δp?

continued

SOLUTIONS

(a) From the point $(1, -3)$ to the point $(4, 5)$, the increments in x and y are

$$\Delta x = 4 - 1 = 3, \quad \Delta y = 5 - (-3) = 8.$$

From the point $(5, 6)$ to the point $(5, 1)$, the increments in x and y are

$$\Delta x = 5 - 5 = 0, \quad \Delta y = 1 - 6 = -5.$$

(b) The increments in depth and pressure are

$$\Delta d = 25 - 20 = 5 \text{ meters}, \quad \Delta p = 3.485 - 2.988 = 0.497 \text{ atmosphere}.$$

Now Try Exercise 1.

Among all functions, *linear functions* are special. They are the functions in which an increment of one unit in one variable always produces exactly the same increment in the other variable. Another way of saying this is that the *ratio of the increments* is constant.

Why Sensitivity?

Sensitivity is a useful way of thinking about the response of one variable to a small change in another variable. The sensitivity to change is measured by the ratio of the increments.

DEFINITION Linear Function

The variable y is a **linear function** of x if the ratio of the increment of y to the increment of x is constant; that is,

$$\frac{\Delta y}{\Delta x} = \text{constant}.$$

In a linear function, the constant ratio of the increments is known as the *rate of change* or the *sensitivity*. The graph of a linear function is a straight line. The constant ratio of the increments is the *slope* of this line.

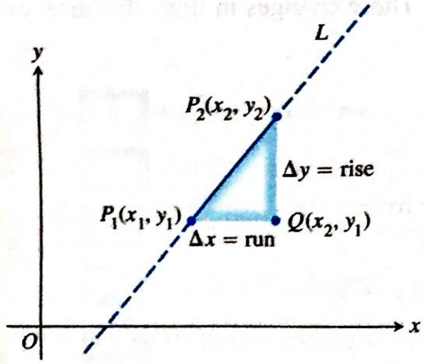


Figure 1.1 The slope of line L is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$.

DEFINITION Slope

Let (x_1, y_1) and (x_2, y_2) be two points on the graph of a linear function. The **slope *m*** of this line is the ratio of the increments; that is, $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

A line with positive slope goes uphill as x increases (Figure 1.1). A line with negative slope goes downhill as x increases. A line with slope zero is horizontal, since $\Delta y = 0$ implies that all of the points have the same y-coordinate. If $\Delta x = 0$, then the x-coordinate never changes and the line is vertical. In this case, we say that the line *has no slope*.

EXAMPLE 2 Using the Slope to Find Coordinates.

Given a line through the point $(2, 3)$ with slope $m = -2$, find the y-coordinate of the point on this line that has x-coordinate 3.6.

SOLUTION

The increment in x is $\Delta x = 3.6 - 2 = 1.6$. Since the ratio of the increments is -2, $\Delta y = -2 \cdot \Delta x = -3.2$.

The y-coordinate is $3 + (-3.2) = -0.2$.

Now Try Exercise 9.

EXAMPLE 3 Finding Additional Values of a Linear Function

The pressure, p, on a diver is a linear function of the depth, d. We know that the pressure at 20 meters is 2.988 atmospheres, and an increase of 5 meters in depth corresponds to an increase of 0.497 atmosphere. Find the pressure at 23 meters, 42 meters, and 16 meters.

SOLUTION

Because this is a linear function, the ratio of the increments is constant:

$$\frac{\Delta p}{\Delta d} = \frac{0.497}{5} = 0.0994 \text{ atmosphere per meter}$$

From $d = 20$ to 23, the change in depth is $\Delta d = 3$, so $\Delta p = 3(0.0994) = 0.2982$. The pressure at 23 m is $2.988 + 0.2982 = 3.2862$ atmospheres.

From $d = 20$ to 42, the change in depth is $\Delta d = 22$, so $\Delta p = 22(0.0994) = 2.1868$. The pressure at 42 m is $2.988 + 2.1868 = 5.1748$ atmospheres.

From $d = 20$ to 16, the change in depth is $\Delta d = -4$, so $\Delta p = -4(0.0994) = -0.3976$. The pressure at 16 m is $2.988 - 0.3976 = 2.5904$ atmospheres. ***Now Try Exercise 13.***

EXAMPLE 4 Modeling a Linear Equation

Using the information given in Example 3, find the linear equation that describes pressure, p, as a linear function of depth, d.

SOLUTION

If p is the pressure at depth d, we get the same constant ratio of increments as the pressure changes from 2.988 to p and the depth changes from 20 to d. We can replace Δp by $p - 2.988$ and Δd by $d - 20$ and solve for p:

$$\frac{p - 2.988}{d - 20} = 0.0994$$

$$p - 2.988 = 0.0994(d - 20) \text{ or } p = 0.0994d + 1$$

Now Try Exercise 17.

Point-Slope Equation of a Linear Function

There are many ways to represent a linear relationship between two variables, and each has a role to play in calculus. The most useful representation will be the *point-slope equation*, built from knowledge of some point on the graph of the function and the value of the constant slope.

If we know some ordered pair (x_1, y_1) that satisfies a linear equation, then the slope m between (x_1, y_1) and any *other* point (x, y) must satisfy the equation $\dfrac{y - y_1}{x - x_1} = m$.

If we multiply this equation by the denominator, we get an equivalent equation that is also valid when (x, y) equals the point (x_1, y_1) itself. This leads to the following definition.

DEFINITION Point-Slope Equation of a Line

The equation

$$y - y_1 = m(x - x_1)$$

is the **point-slope equation** of the line through the point (x_1, y_1) with slope m. We will sometimes find it useful to write this equation in the "calculator-ready" form

$$y = m(x - x_1) + y_1.$$

Zero Denominator Alert

Notice that the equation $\frac{y-3}{x-2} = 5$ does *not* work as an answer in Example 5. Significantly, the graph does not pass through the point $(2, 3)$!

EXAMPLE 5 Using the Point-Slope Equation

Find the point-slope equation of the line with slope 5 that passes through the point $(2, 3)$.

SOLUTION

Letting $m = 5$ and $(x_1, y_1) = (2, 3)$, the point-slope equation is $y - 3 = 5(x - 2)$.

Now Try Exercise 19.

As previously noted, a horizontal line has slope 0. Therefore, the equation of a horizontal line through the point (a, b) has equation $y - b = 0(x - a)$, which simplifies to $y = b$. A vertical line through (a, b) has no slope, and its equation is $x = a$. Horizontal and vertical lines through the point $(2, 3)$ are shown, along with their equations, in Figure 1.2.

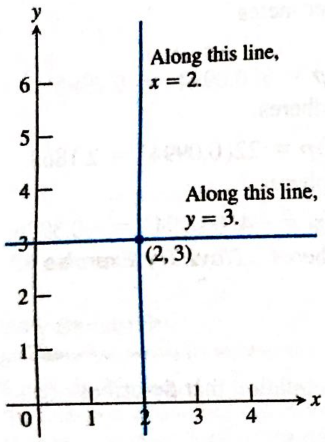


Figure 1.2 The standard equations for the vertical and horizontal lines through the point $(2, 3)$ are $x = 2$ and $y = 3$. (Example 5)

Other Linear Equation Forms

The idea of "slope at a point" is a central theme in calculus, and the point-slope equation of a line conveys the information so perfectly that we will rarely have need in this book for the two other common forms that you may have studied. We list them here under their usual names.

DEFINITION Slope-Intercept Equation and General Linear Equation

The **slope-intercept equation** of a line with slope m that passes through the point $(0, b)$ is

$$y = mx + b.$$

(The number b is called the **y-intercept.** See Figure 1.3.)

A **general linear equation** in x and y has the form

$$Ax + By = C \quad \text{(assuming } A \text{ and } B \text{ are not both } 0\text{)}.$$

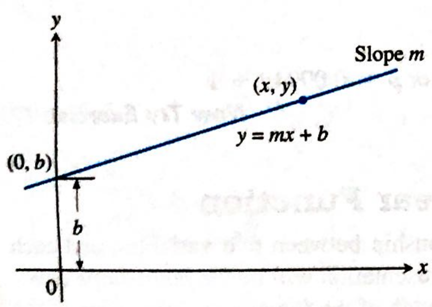


Figure 1.3 A line with slope m and y-intercept b.

Slope-intercept form is useful in modeling real-world problems that have a fixed part and a (proportionately) varying part. For example, $y = mx + b$ could represent the salary of a salesman who earns a base salary of b dollars and a fixed percentage m of his sales of x dollars. It also has the advantage of being *unique* (there is only one possible slope-intercept form for any nonvertical line). The latter quality is quite helpful for authors of multiple-choice tests and textbooks with answer keys, which probably accounts for its dominance in algebra classrooms.

The general linear form is useful in linear algebra. You used it in earlier courses when solving simultaneous linear equations, especially if you used matrices. It is also the only form that accommodates vertical lines, as the other forms require a numerical slope m. We will have very little use for the equations of vertical lines, as they do not represent linear *functions*.

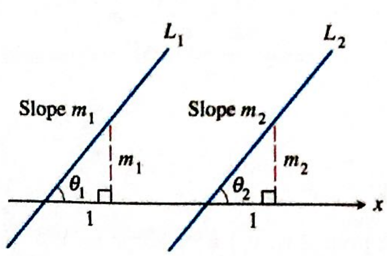


Figure 1.4 If $L_1 \| L_2$, then $\theta_1 = \theta_2$ and $m_1 = m_2$. Conversely, if $m_1 = m_2$, then $\theta_1 = \theta_2$ and $L_1 \| L_2$.

Parallel and Perpendicular Lines

Parallel lines form equal angles with the x-axis (Figure 1.4). Hence, nonvertical parallel lines have the same slope. Conversely, lines with equal slopes form equal angles with the x-axis and are therefore parallel.

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other: If $m_1 = a/b$, then $m_2 = -b/a$.

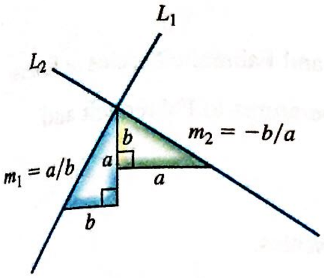


Figure 1.5 A little geometry shows why the slopes of perpendicular lines are negative reciprocals of each other.

In Figure 1.5, the two triangles are congruent right triangles with perpendicular legs of lengths a and b. The angle where lines L_1 and L_2 meet is the sum of the acute angles in this triangle, which is 90°.

EXAMPLE 6 Finding Equations of Lines

Find an equation for:

(a) the line through $(3, 5)$ parallel to the line with equation $y = 2x - 8$;

(b) the line with x-intercept 6 and perpendicular to the line with equation $2x + y = 7$;

(c) the perpendicular bisector of the segment with endpoints $(2, 3)$ and $(6, 17)$.

SOLUTION

(a) The line $y = 2x - 8$ has slope 2, so the line through $(3, 5)$ with the same slope has equation $y - 5 = 2(x - 3)$.

(b) The line $2x + y = 7$ has slope -2, so the line through $(6, 0)$ with negative reciprocal slope has equation $y - 0 = \frac{1}{2}(x - 6)$.

(c) The midpoint of the segment is found by averaging the endpoint coordinates: $\left(\frac{2+6}{2}, \frac{3+17}{2}\right) = (4, 10)$. The slope of the segment is $\frac{17-3}{6-2} = \frac{7}{2}$, so the line through the midpoint and perpendicular to the segment has equation $y - 10 = -\frac{2}{7}(x - 4)$.

Now Try Exercise 29.

Note that we gave the answers in Example 6 in point-slope form, not only because it was easiest, but also because the equation is self-identified as "*the* line through *this* point with *this* slope." It is exactly this interpretation of lines that you will be using in calculus.

If you need to convert from point-slope form to some other form (for example, to select an answer on a multiple-choice test), just do the algebra.

EXAMPLE 7 Doing the Algebra

Match each linear equation below with one of the answers in Example 6.

(a) $x - 2y = 6$ **(b)** $7y + 2x = 78$

(c) $y = 0.5x - 3$ **(d)** $y + 1 = 2x$

SOLUTION

(a) (b) **(b)** (c)

(c) (b) **(d)** (a)

Now Try Exercise 46.

Applications of Linear Functions

When we use variables to represent quantities in the real world, it is not at all unusual for the relationship between them to be linear. As we have seen, the defining characteristic for a linear function is that equal increments in the independent variable must result in equal increments of the dependent variable. In calculus, we will be more interested in extending the slope concept to other functions than in exploring linear functions themselves, but we want to include at least one well-known linear model.

EXAMPLE 8 Temperature Conversion

The relationship between the temperatures on the Celsius and Fahrenheit scales is linear.

(a) Find the linear equations for converting Celsius temperatures to Fahrenheit and Fahrenheit temperatures to Celsius.

(b) Convert 20°C to Fahrenheit and 95°F to Celsius.

(c) Find the unique temperature that is the same in both scales.

SOLUTION

(a) Let C be the Celsius temperature and F the Fahrenheit temperature. Using the freezing point of water and the boiling point of water gives us two reference points in the linear (C, F) relationship: $(0, 32)$ and $(100, 212)$. The slope is $\frac{212 - 32}{100 - 0} = \frac{9}{5}$, and the point-slope equation through $(0, 32)$ is $F - 32 = \frac{9}{5}(C - 0)$. This simplifies to $F = \frac{9}{5}C + 32$.

If we solve this equation for C, we get $C = \frac{5}{9}(F - 32)$, which converts Fahrenheit to Celsius.

(b) When $C = 20$, $F = \frac{9}{5}(20) + 32 = 68$, so 20°C = 68°F.

When $F = 95$, $C = \frac{5}{9}(95 - 32) = 35$, so 95°F = 35°C.

(c) If $F = C$, then $C = \frac{9}{5}C + 32$. Solving for C, we get $C = -40$. When the wind chill is "forty below," it does not matter what scale is being used!

Now Try Exercise 41.

Solving Two Linear Equations Simultaneously

Some exercises in this book will require you to find a unique ordered pair (x, y) that solves two different linear equations simultaneously. There are many ways to do this, and we will review a few of them in Example 9 below. If you need further review, you can consult an algebra textbook.

EXAMPLE 9 Solving Two Linear Equations Simultaneously

Find the unique pair (x, y) that satisfies both of these equations simultaneously:

$$3x - 5y = 39$$
$$2x + 3y = 7$$

SOLUTION 1 (Substitution)

Solve the first equation for x to find that $x = \frac{5}{3}y + 13$. Substitute the expression into the second equation to get $2\left(\frac{5}{3}y + 13\right) + 3y = 7$. Solve this equation to get $y = -3$. Finally, plug $y = -3$ into either original equation to get $x = 8$. The answer is $(8, -3)$.

continued

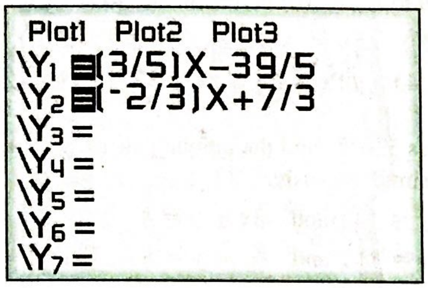
SOLUTION 2 (Elimination)

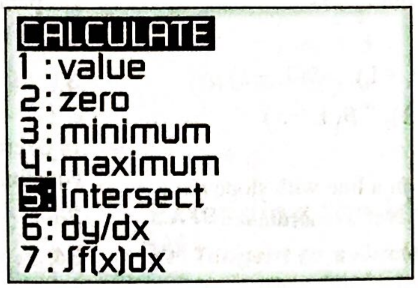
$$\begin{matrix} 3x - 5y = 39 & & 6x - 10y = 78 \\ & \Rightarrow & \\ 2x + 3y = 7 & & 6x + 9y = 21 \end{matrix}$$

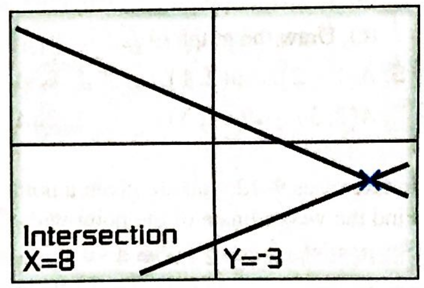
Subtract the bottom equation from the top to "eliminate" the x variable and get $-19y = 57$. Solve this equation to get $y = -3$ and proceed as in Solution 1.

SOLUTION 3 (Graphical)

Graph the two lines and use the "intersect" command on your graphing calculator:

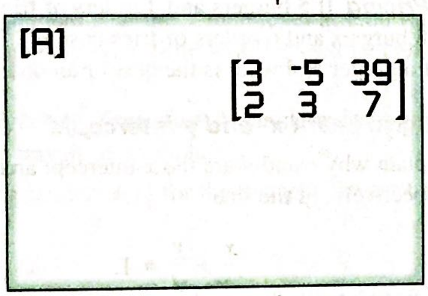


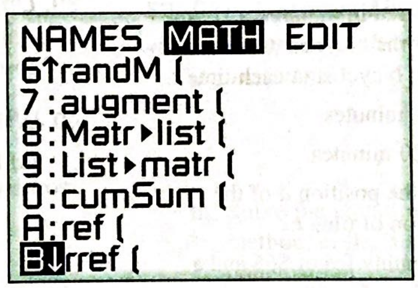


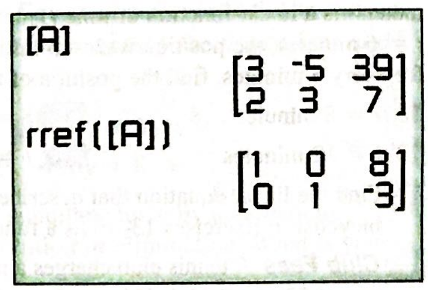


SOLUTION 4 (Matrix Manipulation)

Enter the coefficients into a 2×3 matrix and use your calculator to find the row-reduced echelon form (rref). The values of x and y will appear in the third column.







Now Try Exercise 33.

There is much more that could be said about solving simultaneous linear equations, but that is for another course. If you can use one or more of these methods to solve two linear equations in two variables, that will suffice for now.

Quick Review 1.1 *(For help, go to Section 1.1.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

1. Find the value of y that corresponds to $x = 3$ in $y = -2 + 4(x - 3)$.

2. Find the value of x that corresponds to $y = 3$ in $y = 3 - 2(x + 1)$.

In Exercises 3 and 4, find the value of m that corresponds to the values of x and y.

3. $x = 5, \quad y = 2, \quad m = \dfrac{y - 3}{x - 4}$

4. $x = -1, \quad y = -3, \quad m = \dfrac{2 - y}{3 - x}$

In Exercises 5 and 6, determine whether the ordered pair is a solution to the equation.

5. $3x - 4y = 5$
(a) $(2, 1/4)$ **(b)** $(3, -1)$

6. $y = -2x + 5$
(a) $(-1, 7)$ **(b)** $(-2, 1)$

In Exercises 7 and 8, find the distance between the points.

7. $(1, 0), \quad (0, 1)$

8. $(2, 1), \quad (1, -1/3)$

In Exercises 9 and 10, solve for y in terms of x.

9. $4x - 3y = 7$

10. $-2x + 5y = -3$

Section 1.1 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

In Exercises 1–4, find the coordinate increments from A to B.

1. $A(1, 2)$, $B(-1, -1)$
2. $A(-3, 2)$, $B(-1, -2)$
3. $A(-3, 1)$, $B(-8, 1)$
4. $A(0, 4)$, $B(0, -2)$

In Exercises 5–8, let L be the line determined by points A and B.

(a) Plot A and B.
(b) Find the slope of L.
(c) Draw the graph of L.

5. $A(1, -2)$, $B(2, 1)$
6. $A(-2, -1)$, $B(1, -2)$
7. $A(2, 3)$, $B(-1, 3)$
8. $A(1, 2)$, $B(1, -3)$

In Exercises 9–12, you are given a point P on a line with slope m. Find the y-coordinate of the point with the given x-coordinate.

9. $P(3, 5)$ $m = 2$ $x = 4.5$
10. $P(-2, 1)$ $m = 3$ $x = 2$
11. $P(3, 2)$ $m = -3$ $x = 5$
12. $P(-1, -2)$ $m = 0.8$ $x = 1$

In Exercises 13–17, the position d of a bicyclist (measured in kilometers) is a linear function of time t (measured in minutes). At time $t = 6$ minutes, the position is $d = 5$ km. If the bicyclist travels 2 km for every 5 minutes, find the position of the bicyclist at each time t.

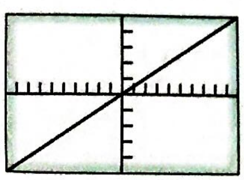
13. $t = 8$ minutes
14. $t = 3$ minutes
15. $t = 12$ minutes
16. $t = 20$ minutes
17. Find the linear equation that describes the position d of the bicyclist in Exercises 13–16 as a function of time t.

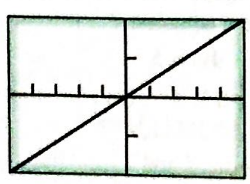
18. ***Club Fees*** A tennis club charges a monthly fee of \$65 and a rate of \$20 for each half-hour of court time. Find the linear equation that gives the total monthly fee F for a club member who accumulates t hours of court time during the month.

In Exercises 19–22, write the point-slope equation for the line through the point P with slope m.

19. $P(1, 1)$, $m = 1$
20. $P(-1, 1)$, $m = -1$
21. $P(0, 3)$, $m = 2$
22. $P(-4, 0)$, $m = -2$

In Exercises 23 and 24, the line contains the origin and the point in the upper right corner of the grapher screen. Write an equation for the line.

23.

[−10, 10] by [−25, 25]

24.

[−5, 5] by [−2, 2]

In Exercises 25–28, find the **(a)** slope and **(b)** y-intercept, and **(c)** graph the line.

25. $3x + 4y = 12$
26. $x + y = 2$
27. $\frac{x}{3} + \frac{y}{4} = 1$
28. $y = 2x + 4$

In Exercises 29–32, write an equation for the line through P that is **(a)** parallel to L, and **(b)** perpendicular to L.

29. $P(0, 0)$, $L: y = -x + 2$
30. $P(-2, 2)$, $L: 2x + y = 4$
31. $P(-2, 4)$, $L: x = 5$
32. $P(-1, 1/2)$, $L: y = 3$

In Exercises 33–38, find the unique pair (x, y) that satisfies both equations simultaneously.

33. $x - 2y = 13$ and $3x + y = 4$
34. $2x + y = 11$ and $6x - y = 5$
35. $20x + 7y = 22$ and $y - 5x = 11$
36. $2y - 5x = 0$ and $4x + y = 26$
37. $4x - y = 4$ and $14x + 3y = 1$
38. $3x + 2y = 4$ and $12x - 5y = 3$

39. ***Unit Pricing*** If 5 burgers and 4 orders of fries cost \$30.76, while 8 burgers and 6 orders of fries cost \$48.28, what is the cost of a burger and what is the cost of an order of fries?

40. Writing to Learn *x- and y-intercepts*

(a) Explain why c and d are the x-intercept and y-intercept, respectively, of the line

$$\frac{x}{c} + \frac{y}{d} = 1.$$

(b) How are the x-intercept and y-intercept related to c and d in the line

$$\frac{x}{c} + \frac{y}{d} = 2?$$

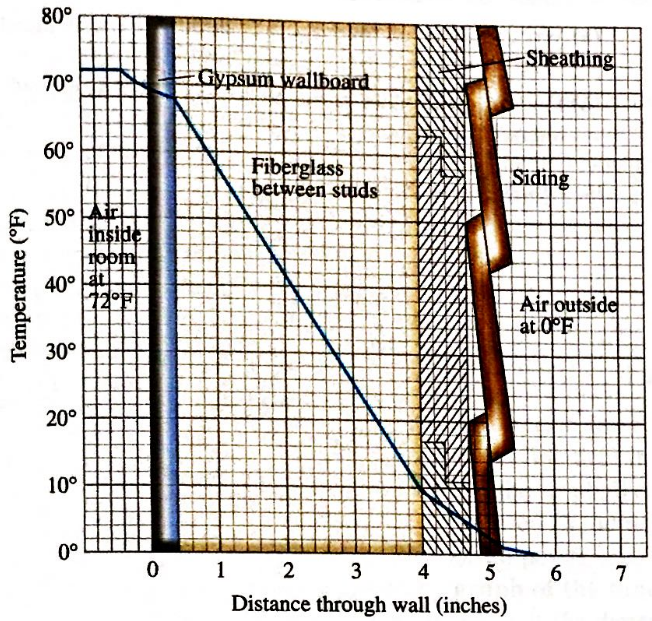
41. ***Parallel and Perpendicular Lines*** For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$ **(a)** parallel? **(b)** perpendicular?

Group Activity In Exercises 42–44, work in groups of two or three to solve the problem.

42. ***Insulation*** By measuring slopes in the figure below, find the temperature change in degrees per inch for the following materials.

(a) gypsum wallboard
(b) fiberglass insulation
(c) wood sheathing

(d) **Writing to Learn** Which of the materials in (a)–(c) is the best insulator? the poorest? Explain.



43. ***For the Birds*** The level of seed in Bruce's bird feeder declines linearly over time. If the feeder is filled to the 12-inch level at 10:00 AM and is at the 7-inch level at 2:00 PM the same day, at approximately what time will the seed be completely gone?

44. ***Modeling Distance Traveled*** A car starts from point P at time $t = 0$ and travels at 45 mph.

(a) Write an expression $d(t)$ for the distance the car travels from P.

(b) Graph $y = d(t)$.

(c) What is the slope of the graph in (b)? What does it have to do with the car?

(d) **Writing to Learn** Create a scenario in which t could have negative values.

(e) **Writing to Learn** Create a scenario in which the y-intercept of $y = d(t)$ could be 30.

Standardized Test Questions

45. **True or False** The slope of a vertical line is zero. Justify your answer.

46. **True or False** The slope of a line perpendicular to the line $y = mx + b$ is $1/m$. Justify your answer.

47. **Multiple Choice** Which of the following is an equation of the line through $(-3, 4)$ with slope $1/2$?

(A) $y - 4 = \frac{1}{2}(x + 3)$ (B) $y + 3 = \frac{1}{2}(x - 4)$

(C) $y - 4 = -2(x + 3)$ (D) $y - 4 = 2(x + 3)$

(E) $y + 3 = 2(x - 4)$

48. **Multiple Choice** Which of the following is an equation of the vertical line through $(-2, 4)$?

(A) $y = 4$ (B) $x = 2$ (C) $y = -4$

(D) $x = 0$ (E) $x = -2$

49. **Multiple Choice** Which of the following is the x-intercept of the line $y = 2x - 5$?

(A) $x = -5$ (B) $x = 5$ (C) $x = 0$

(D) $x = 5/2$ (E) $x = -5/2$

50. **Multiple Choice** Which of the following is an equation of the line through $(-2, -1)$ parallel to the line $y = -3x + 1$?

(A) $y = -3x + 5$ (B) $y = -3x - 7$ (C) $y = \frac{1}{3}x - \frac{1}{3}$

(D) $y = -3x + 1$ (E) $y = -3x - 4$

Extending the Ideas

51. ***Tangent to a Circle*** A circle with radius 5 centered at the origin passes through the point $(3, 4)$. Find an equation for the line that is tangent to the circle at that point.

52. ***Knowing Your Rights*** The vertices of triangle ABC have coordinates $A(-3, 10)$, $B(1, 3)$, and $C(15, 11)$. Prove that it is a right triangle. Which side is the hypotenuse?

53. ***Simultaneous Linear Equations Revisited*** The two linear equations shown below are said to be *dependent and inconsistent*:

$$3x - 5y = 3$$
$$-9x + 15y = 8$$

(a) Solve the equations simultaneously by an algebraic method, either substitution or elimination. What is your conclusion?

(b) What happens if you use a graphical method?

(c) **Writing to Learn** Explain in algebraic and graphical terms what happens when two linear equations are dependent and inconsistent.

54. ***Simultaneous Linear Equations Revisited Again*** The two linear equations shown below are said to be *dependent and consistent*:

$$2x - 5y = 3$$
$$6x - 15y = 9$$

(a) Solve the equations simultaneously by an algebraic method, either substitution or elimination. What is your conclusion?

(b) What happens if you use a graphical method?

(c) **Writing to Learn** Explain in algebraic and graphical terms what happens when two linear equations are dependent and consistent.

55. ***Parallelogram*** Three different parallelograms have vertices at $(-1, 1)$, $(2, 0)$, and $(2, 3)$. Draw the three and give the coordinates of the missing vertices.

56. ***Parallelogram*** Show that if the midpoints of consecutive sides of any quadrilateral are connected, the result is a parallelogram.

57. ***Tangent Line*** Consider the circle of radius 5 centered at $(1, 2)$. Find an equation of the line tangent to the circle at the point $(-2, 6)$.

58. **Group Activity** ***Distance from a Point to a Line*** This activity investigates how to find the distance from a point $P(a, b)$ to a line $L: Ax + By = C$.

(a) Write an equation for the line M through P perpendicular to L.

(b) Find the coordinates of the point Q in which M and L intersect.

(c) Find the distance from P to Q.